Heavy quarkonia at finite temperature: The EFT approach

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Outline

- Motivation
- Introduction to Effective Field Theories at T=0
- The EFT approach for quarkonia at finite temperature
- Conclusions

Motivations

- Over the past two decades many Effective Field Theories of QCD have been developed
  - ChPT for the study of low-energy hadronic physics
  - Non-Relativistic QCD / potential NRQCD for heavy quarkonium physics
  - SCET for jet physics
  - At finite T EQCD/MQCD, HTL
  - ...

Goal

- Our goal is then to extend the well-established T=0 EFT formalism for heavy quarkonia to the finite temperature situation
  - EFT help understanding and disentangling contribution from the various scales
  - Systematic, non model-based approach to the potential

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Wilson coefficient

Low-energy operator/large scale

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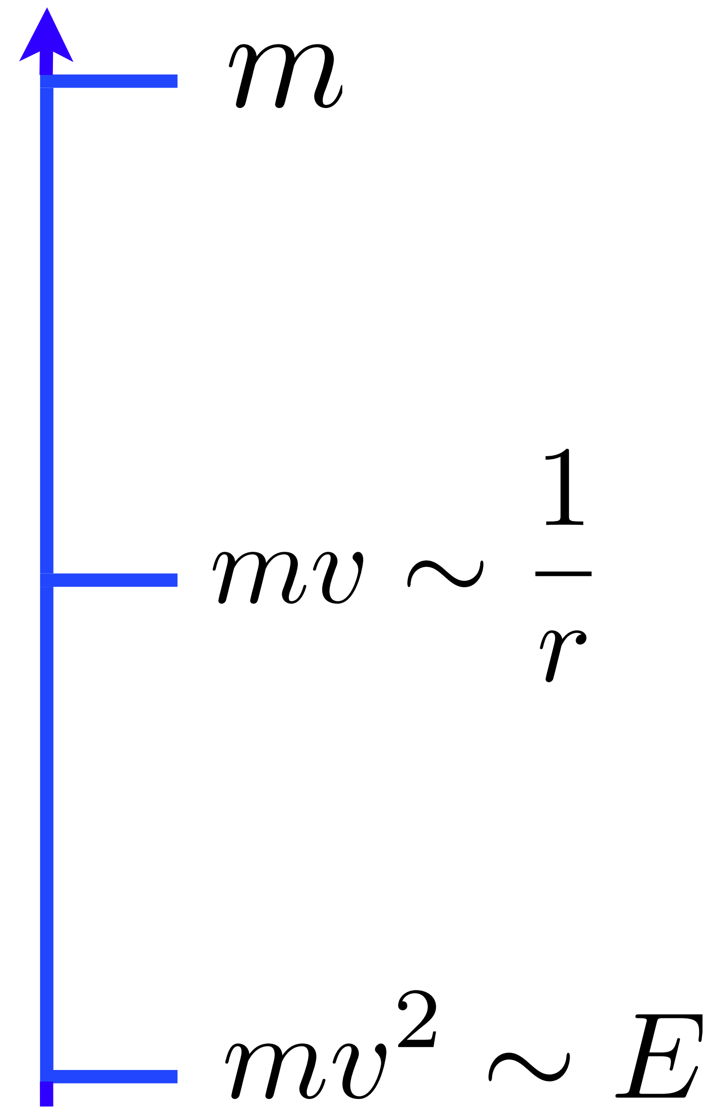
- The procedure can be iterated $\ldots \ll \mu_2 \ll \Lambda_2 \ll \mu_1 \ll \Lambda_1$

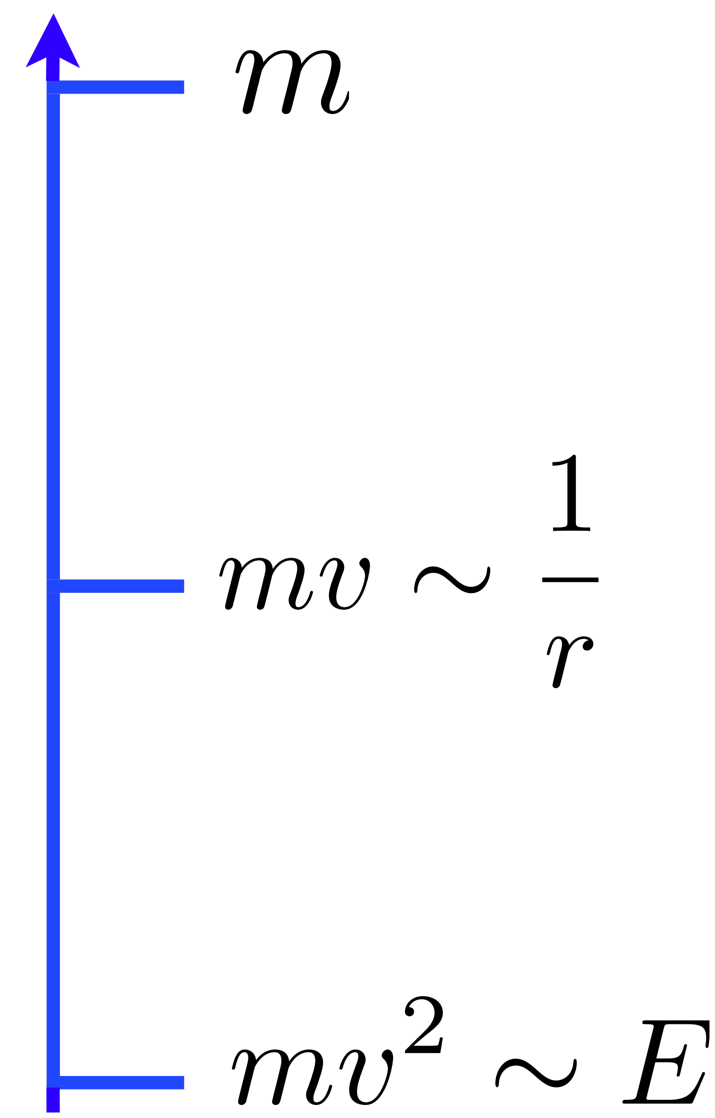
T=0 NR EFTs: a short intro

$Q\overline{Q}$



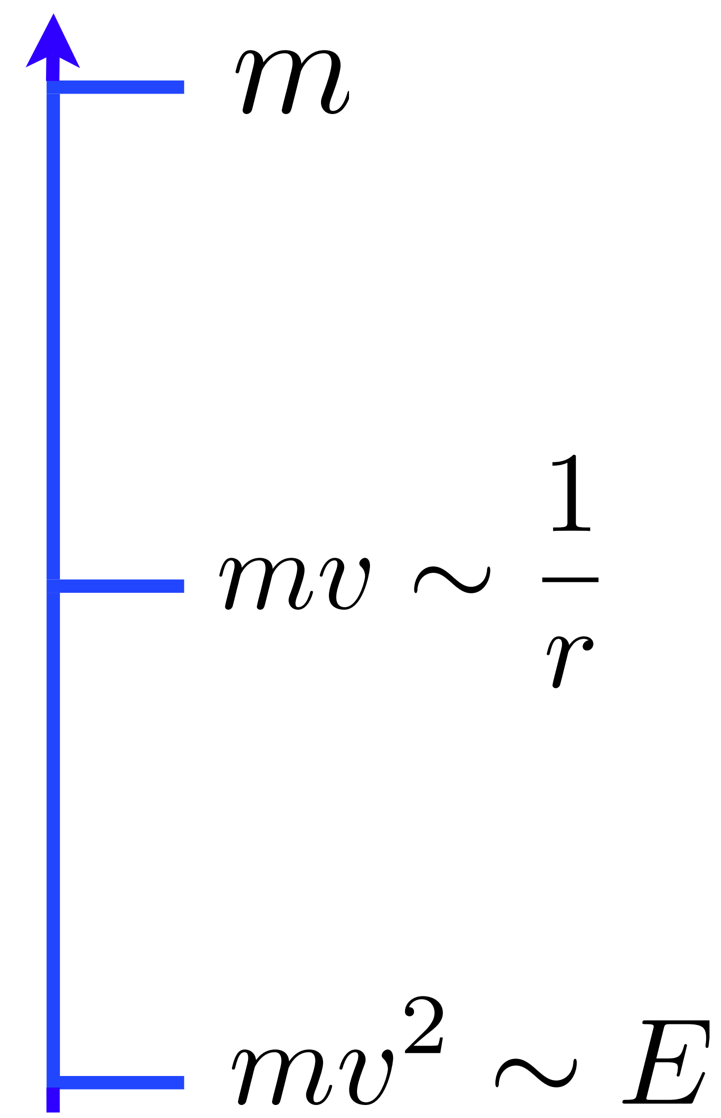
T=0 NR EFTs: a short intro

- Non-relativistic $Q\overline{Q}$ bound states are characterized by the hierarchy of the mass, energy and momentum scales

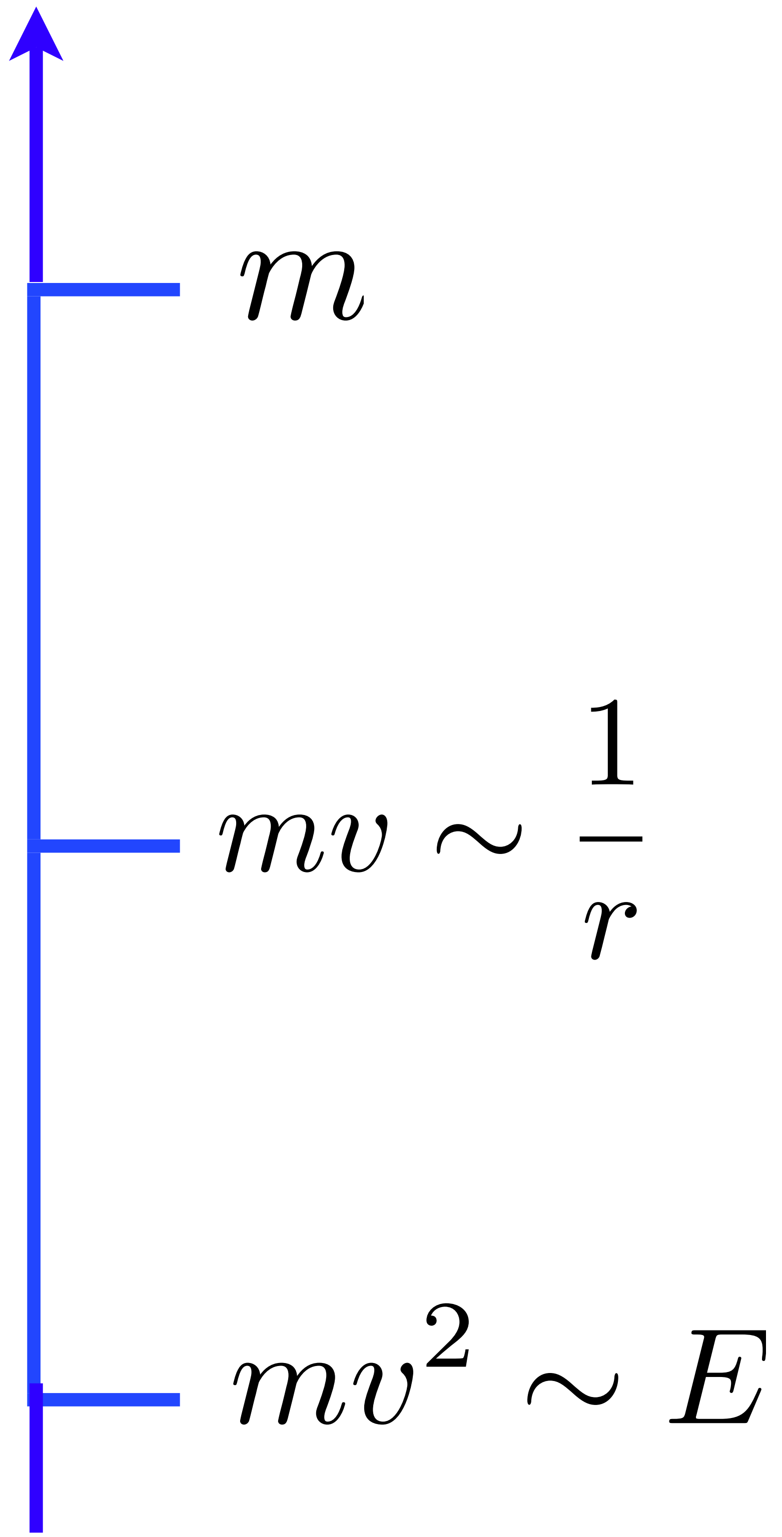


T=0 NR EFTs: a short intro

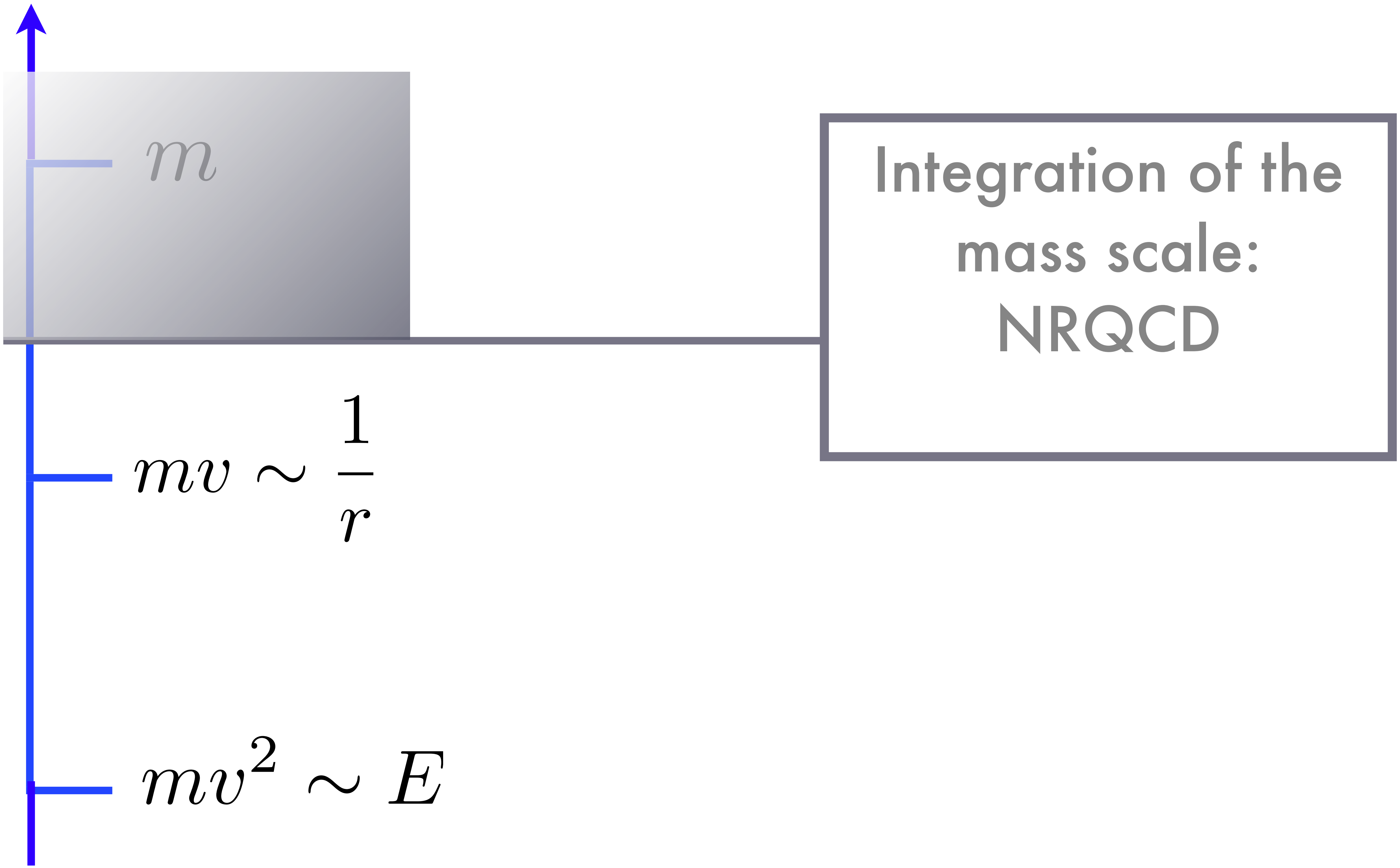
- Non-relativistic $Q\overline{Q}$ bound states are characterized by the hierarchy of the mass, energy and momentum scales
- One can then expand observables in terms of the ratio of the scales and construct a *hierarchy of EFTs* that are equivalent to QCD order-by-order in the expansion parameter



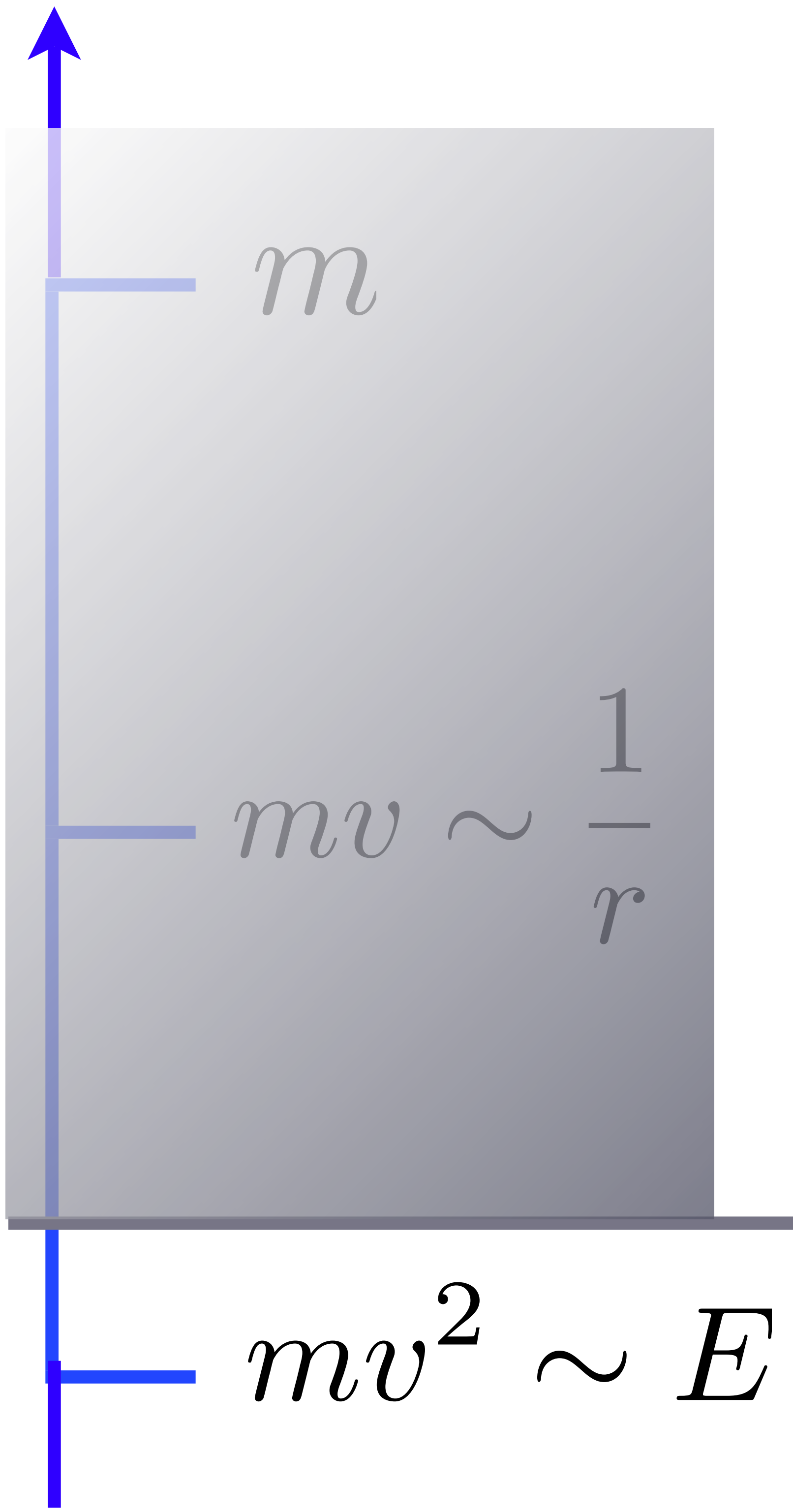
T=0 Scales



T=0 Scales

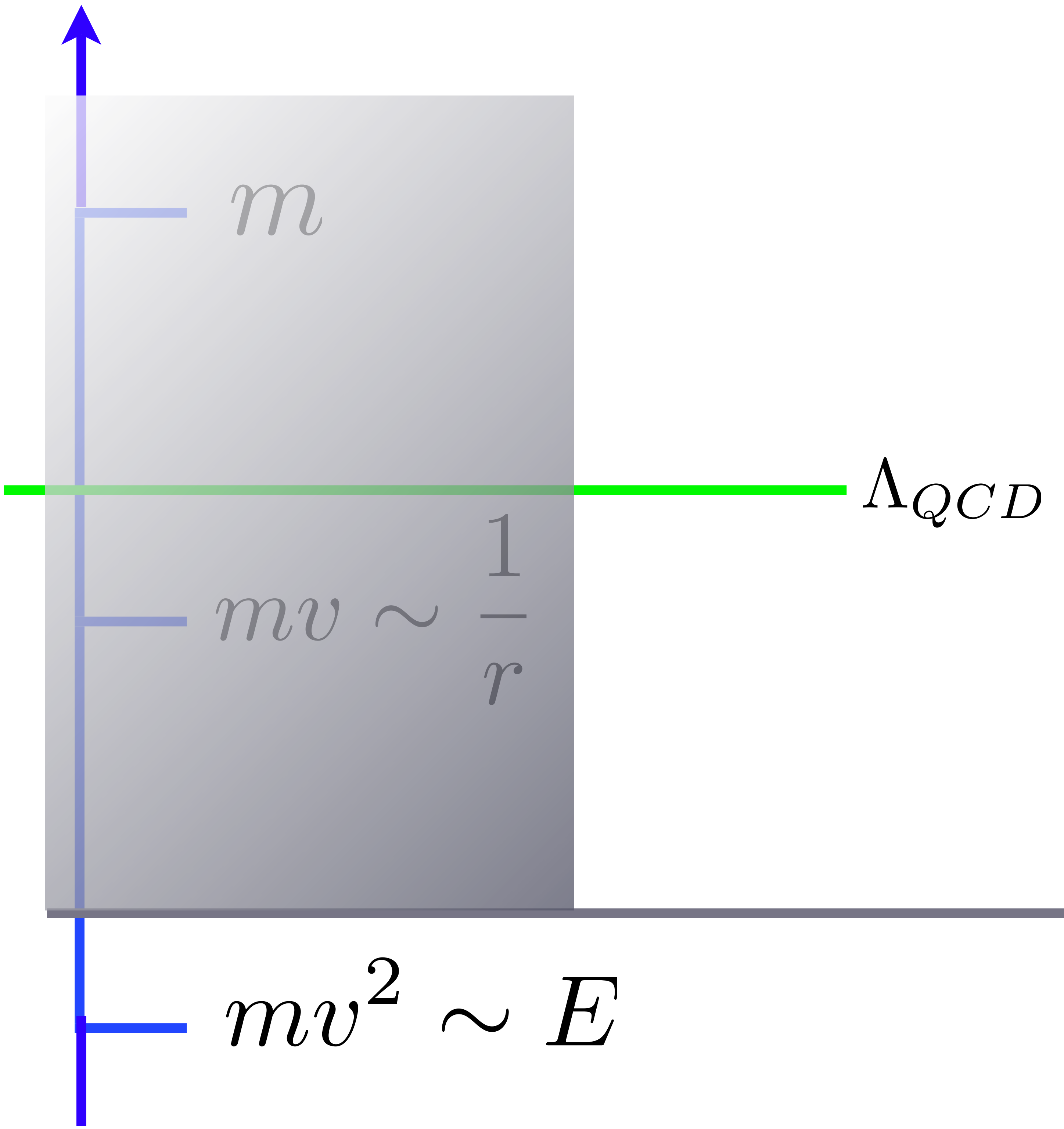


T=0 Scales



Integration of the soft (momentum transfer) scale: pNRQCD

T=0 Scales



Integration of the soft (momentum transfer) scale: pNRQCD

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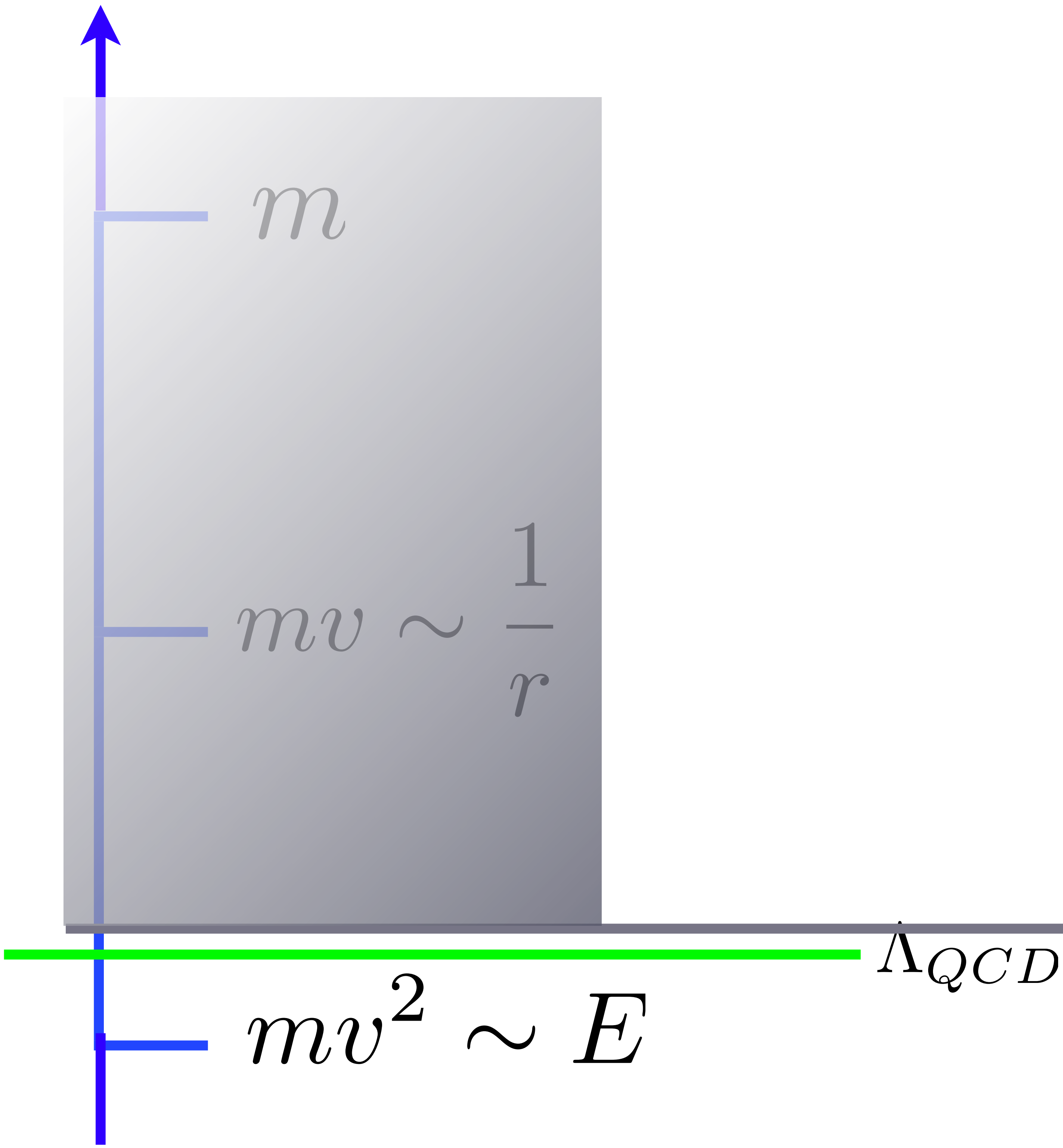

Integration of the soft (momentum transfer) scale: pNRQCD

Weakly coupled pNRQCD

$$\mathcal{L} = \mathrm{Tr}\left[S^\dagger\left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s\right)S + O^\dagger\left(iD_o - \frac{\mathbf{p}^2}{m} - V_0\right)O\right]$$

$$+gV_A(r)\mathrm{Tr}\left[O^\dagger\mathbf{r}\cdot\mathbf{E}S + S^\dagger\mathbf{r}\cdot\mathbf{E}O\right] + g\frac{V_B(r)}{2}\mathrm{Tr}\left[O^\dagger\mathbf{r}\cdot\mathbf{E}O + O^\dagger O\mathbf{r}\cdot\mathbf{E}\right] + \ldots$$

- Degrees of freedom: $Q\overline{Q}$ states with energy $E \sim \Lambda_{QCD}, mv^2$ and momentum $p \lesssim mv$
  Singlet and octet color states
- US gluons with energy / momentum $\lesssim mv$
- Expansion in α_s , $\frac{1}{m}$ and r
- Potential is a Wilson coefficient, receives contributions from all higher scales

Thermodynamical scales

- The thermal medium introduces new scales in the physical problem
  - The temperature
  - The electric screening scale (Debye mass)
  - The magnetic screening scale (magnetic mass)
- In the weak coupling assumption these scales develop a hierarchy

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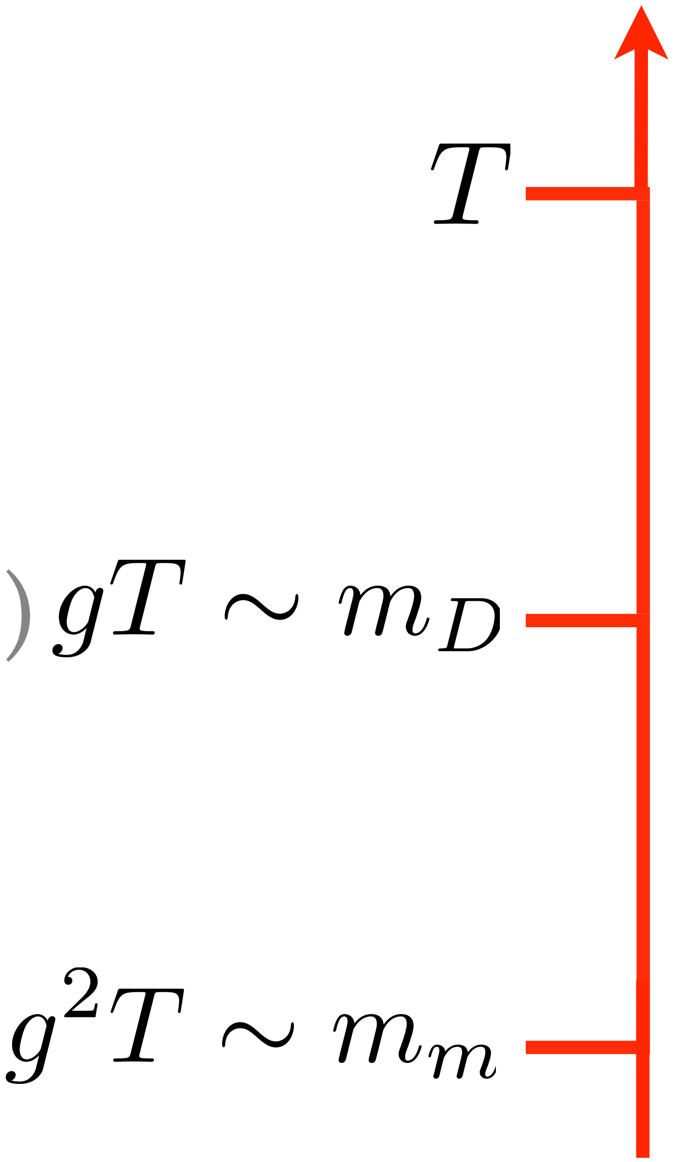


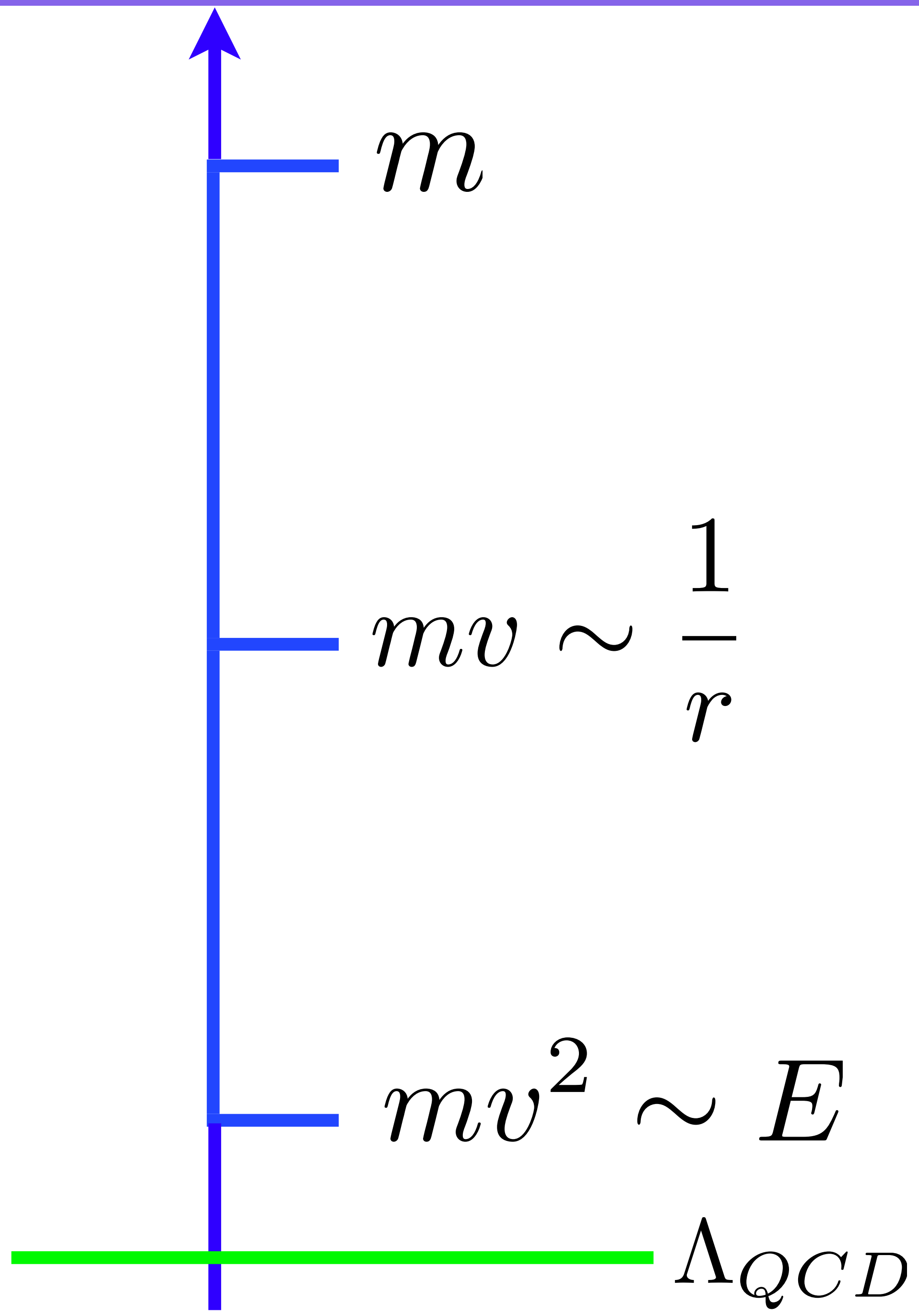
Scales of the problem

$$m$$

$$mv \sim \frac{1}{r}$$

$$mv^2 \sim E$$

Scales of the problem



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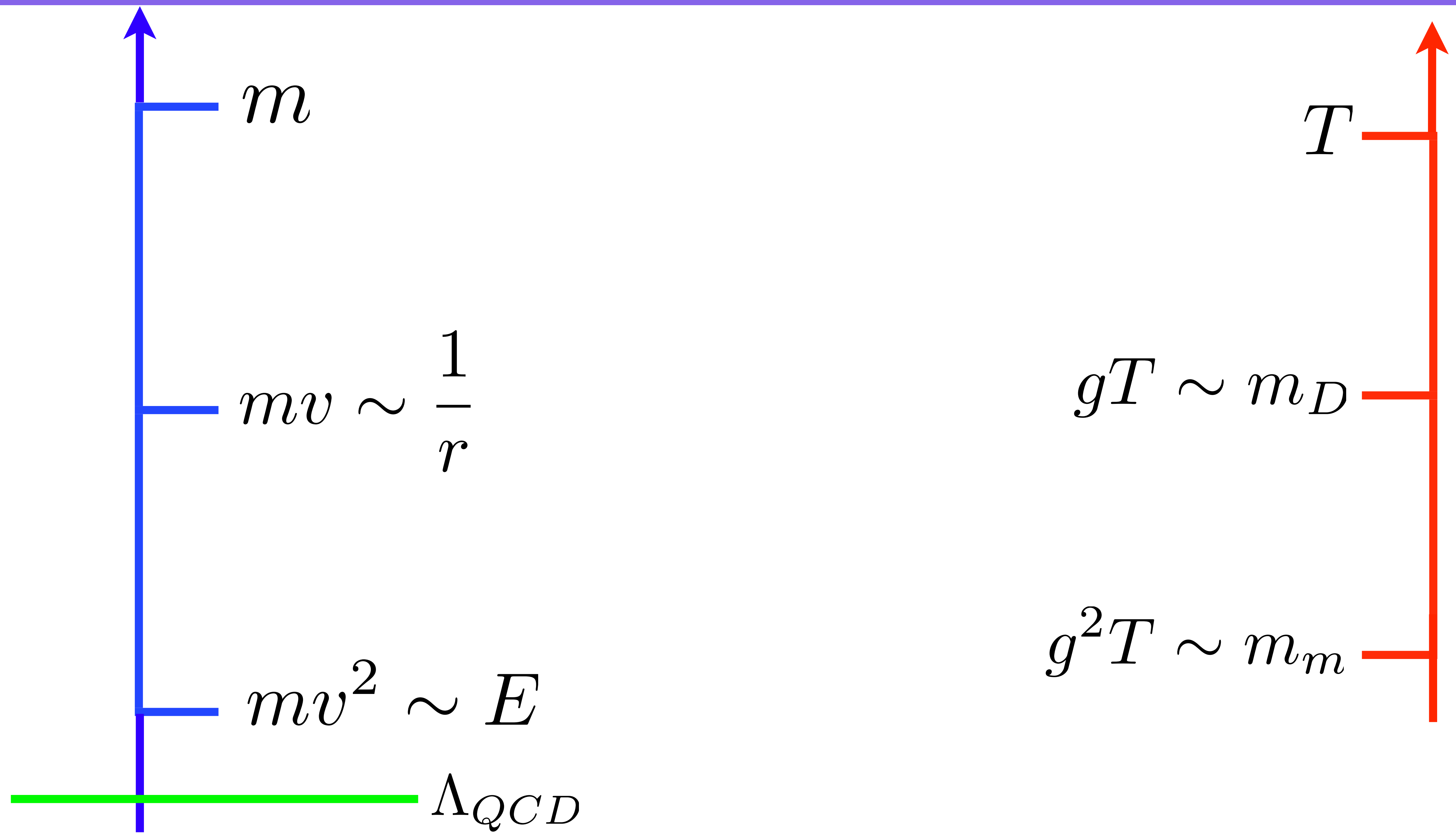

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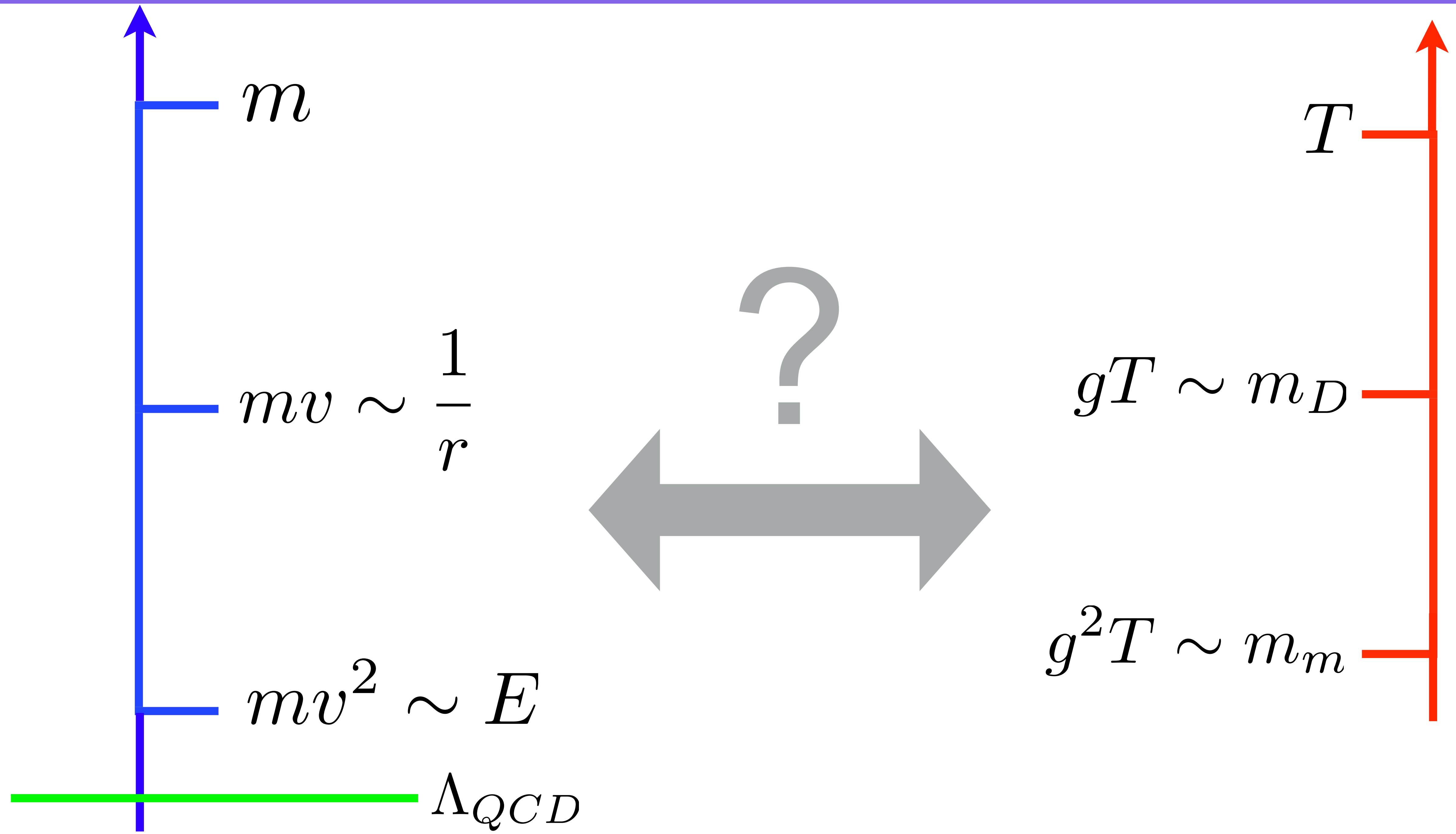

Scales of the problem

- In our work various possibilities have been studied, from $T \ll E$ to $m \gg T \gg 1/r \sim m_D$
- In the regime $T \gg \frac{1}{r} \sim m_D$
  we reobtain the result of Laine et al 2007

- Here we illustrate the intermediate case
  $$m \gg 1/r \gg T \gg m_D \gg E$$
  Brambilla JG Petreczky Vairo 2008

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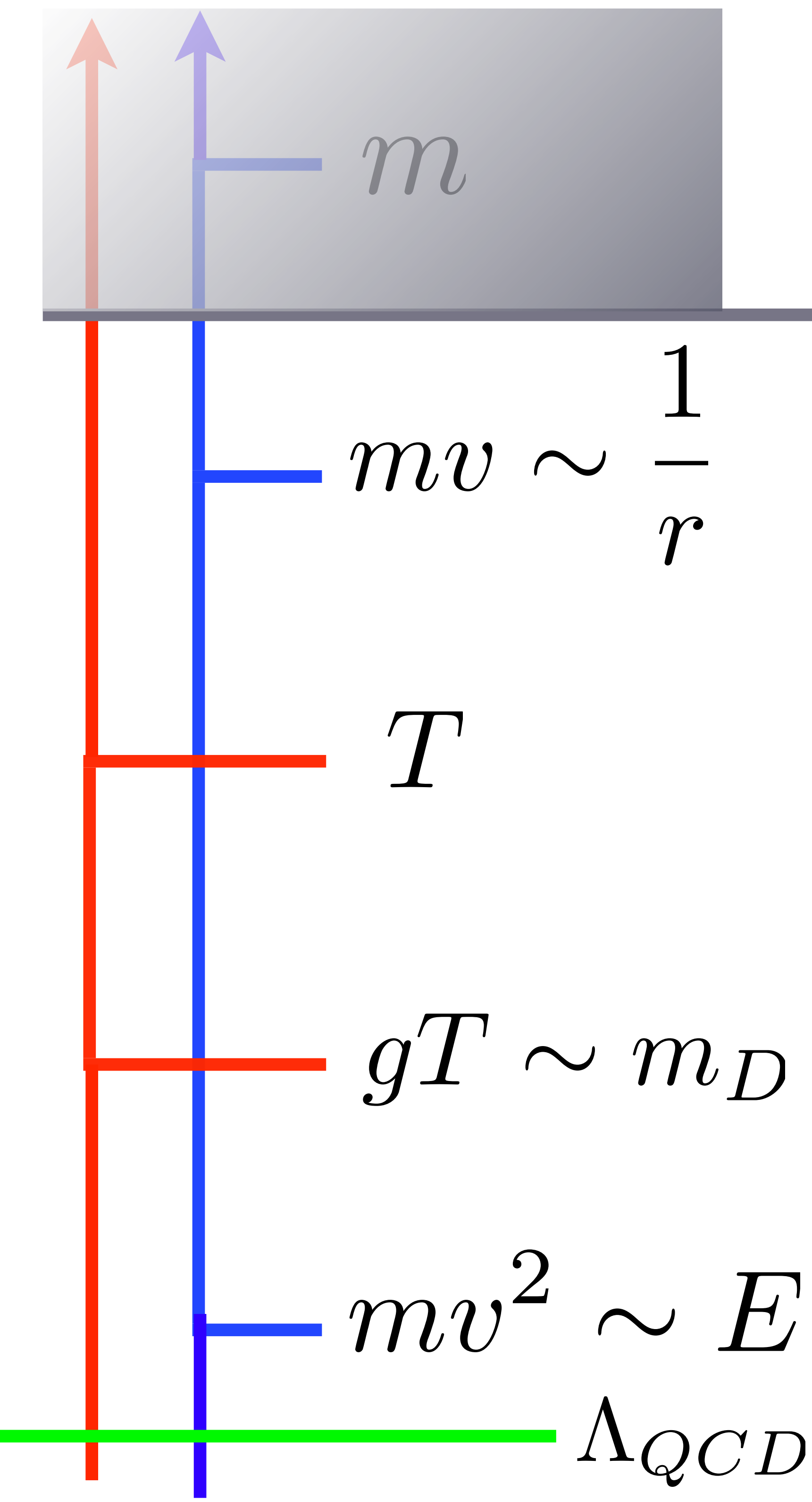
$$V_{\mathrm{HTL}}(r) = -\alpha_s C_F \left(\frac{e^{-m_D r}}{r} - i \frac{2T}{m_D r} f(m_D r) \right)$$

- Here we illustrate the intermediate case

$$m \gg 1/r \gg T \gg m_D \gg E$$

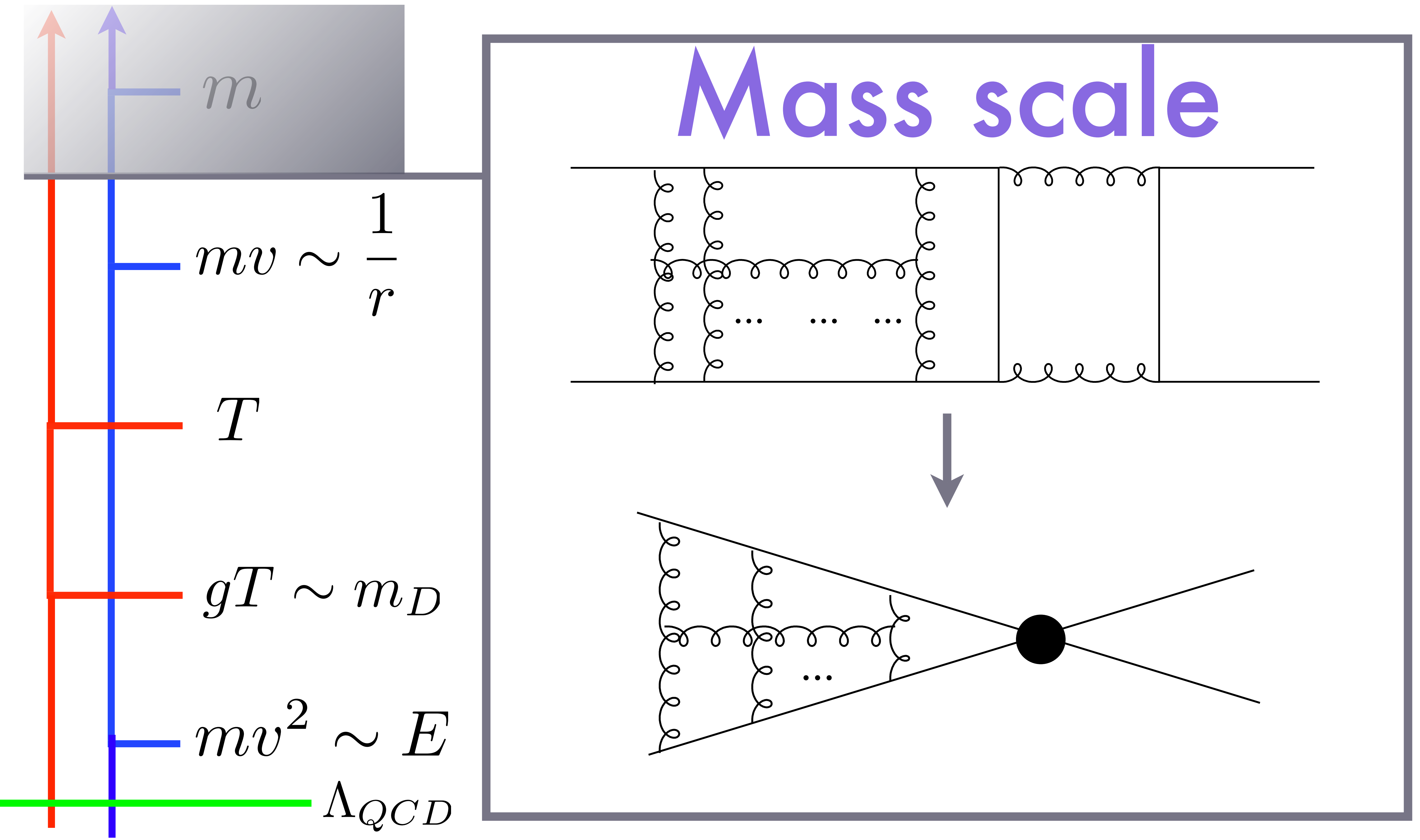
Brambilla JG Petreczky Vairo 2008



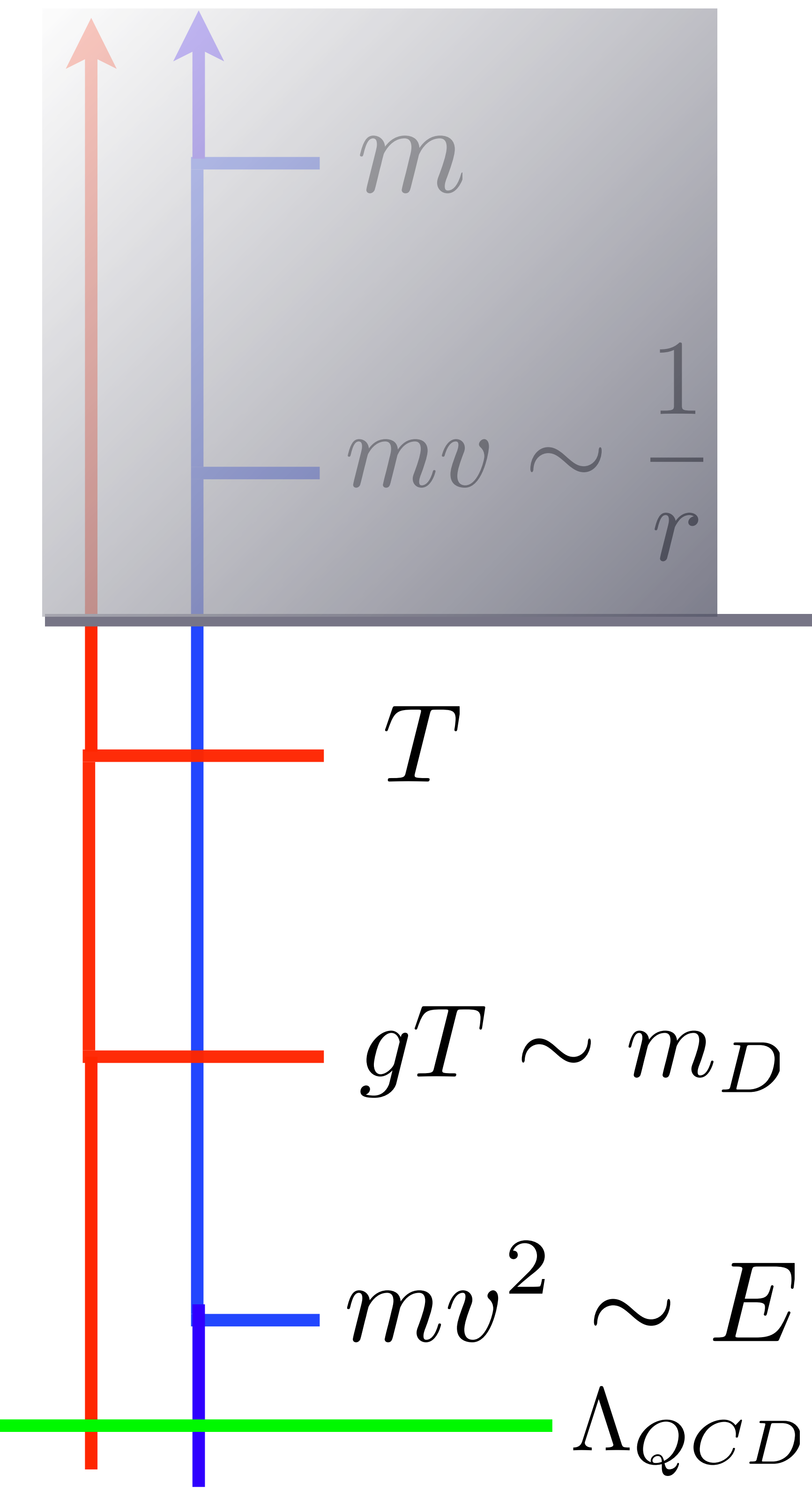
Mass scale

- QCD $\Rightarrow$NRQCD
- We only consider the leading term $\left(\frac{1}{m}\right)^0$, corresponding to treating heavy quarks/ antiquarks as static sources
- So far everything goes exactly as in the T=0 case

Caswell Lepage 86



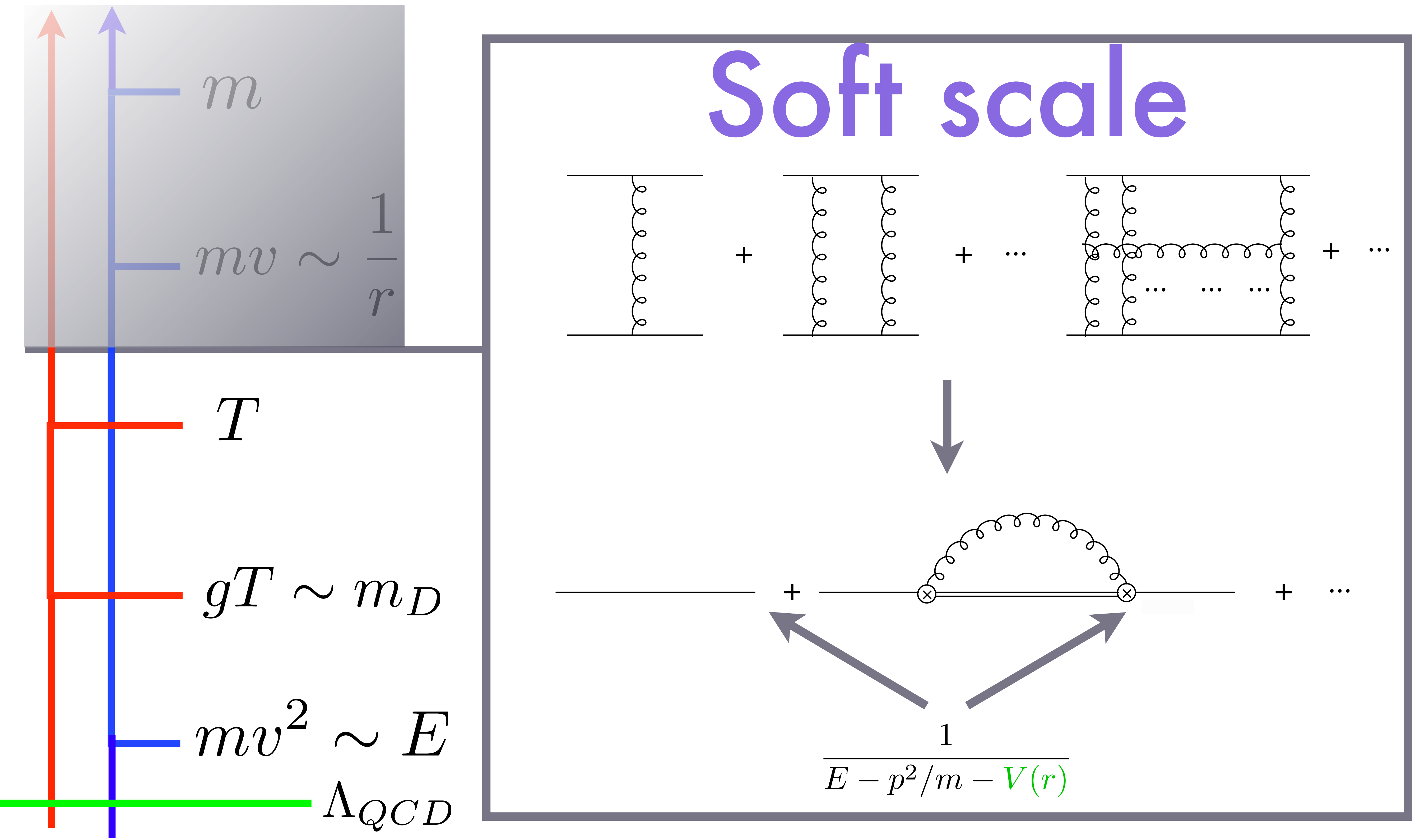
m
$mv \sim \frac{1}{r}$
T
$gT \sim m_D$
$mv^2 \sim E$
Λ_{QCD}
Mass scale
...



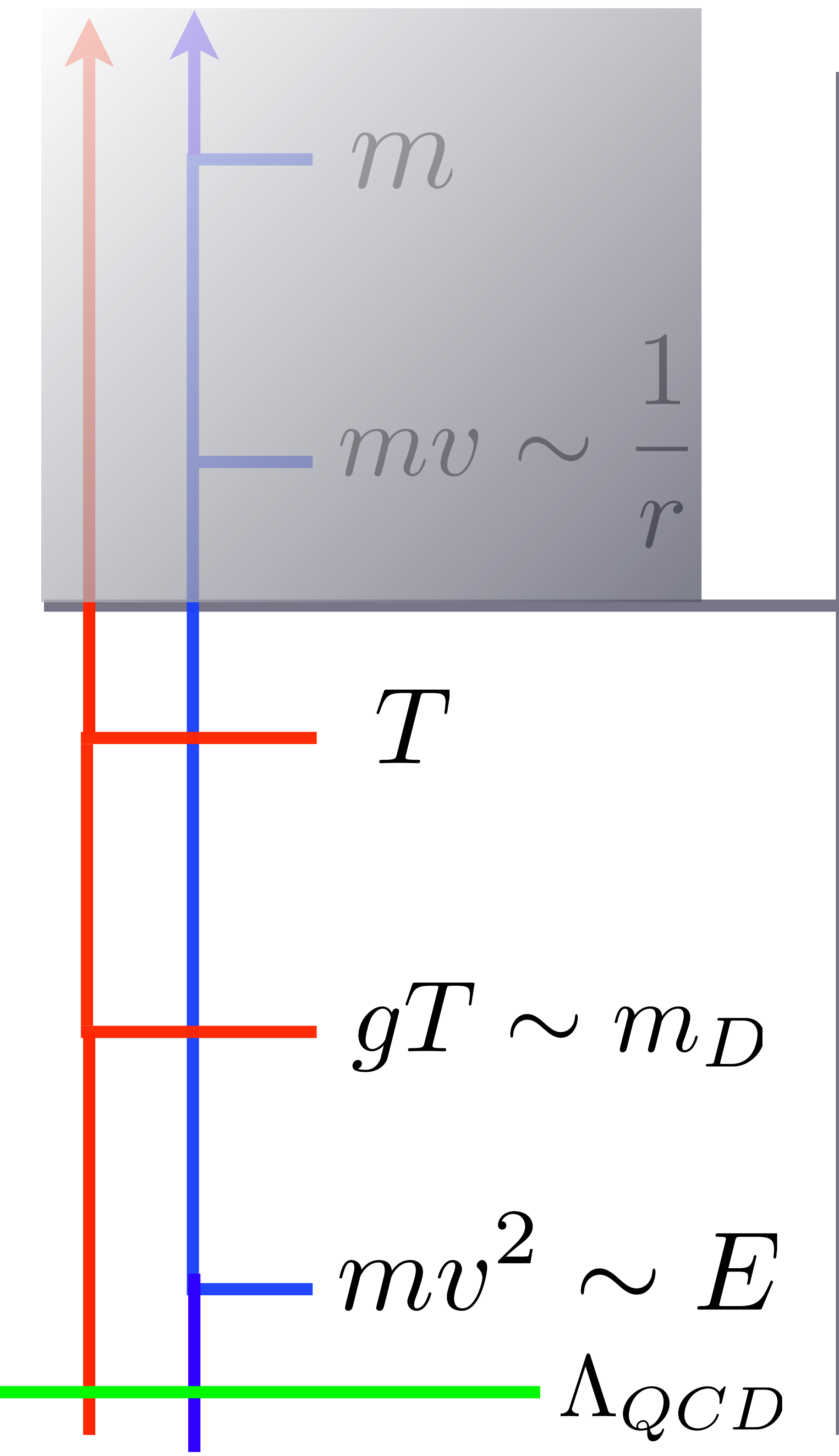
Soft scale

- NRQCD ⇒pNRQCD
- Integrating out the soft modes causes the singlet and octet potentials to appear

Pineda Soto 98
Brambilla Pineda Soto Vairo 99



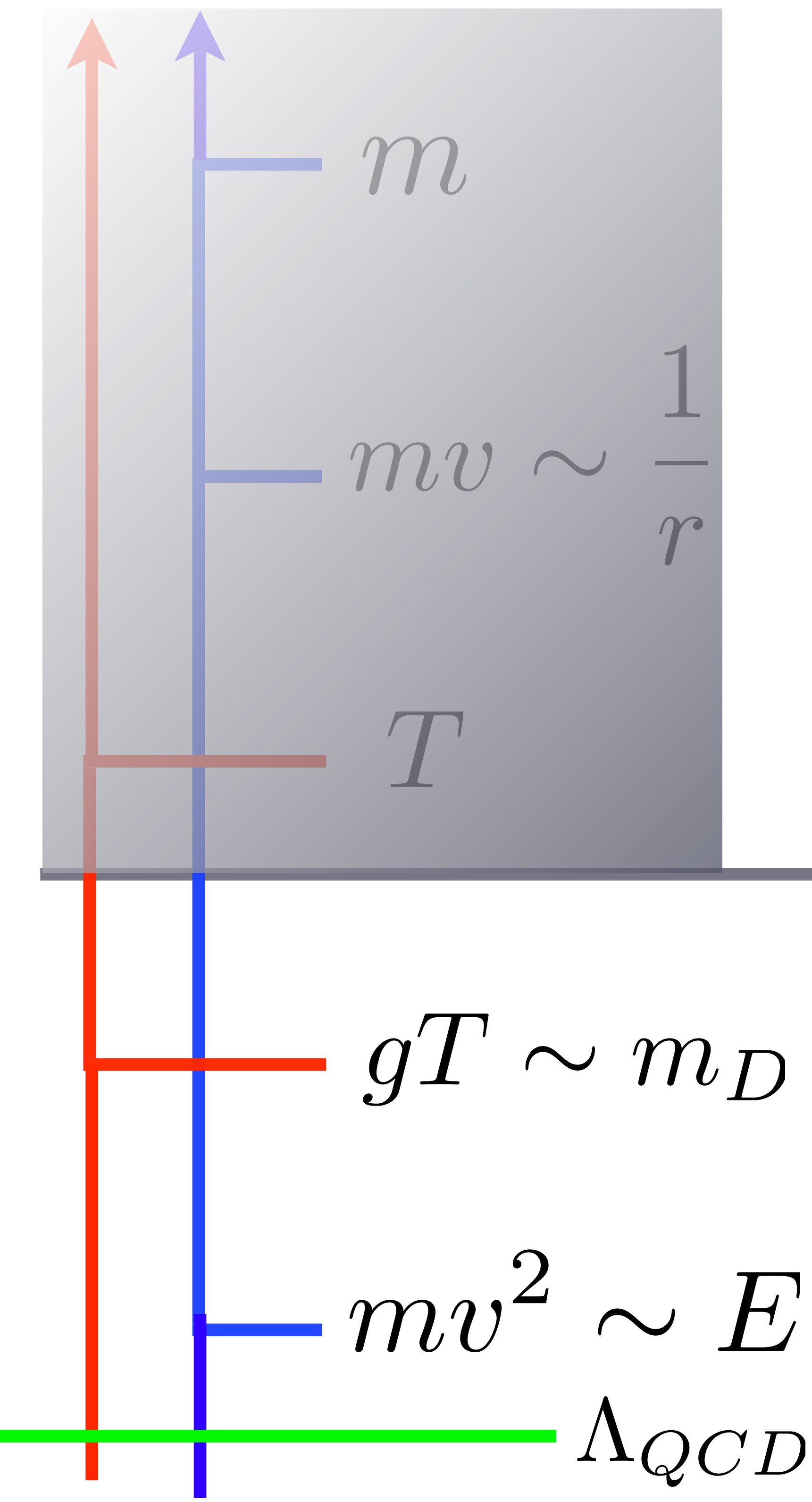
Soft scale
m
$mv \sim \frac{1}{r}$
T
$gT \sim m_D$
$mv^2 \sim E$
Λ_{QCD}
$\frac{1}{E - p^2/m - V(r)}$



The static potential

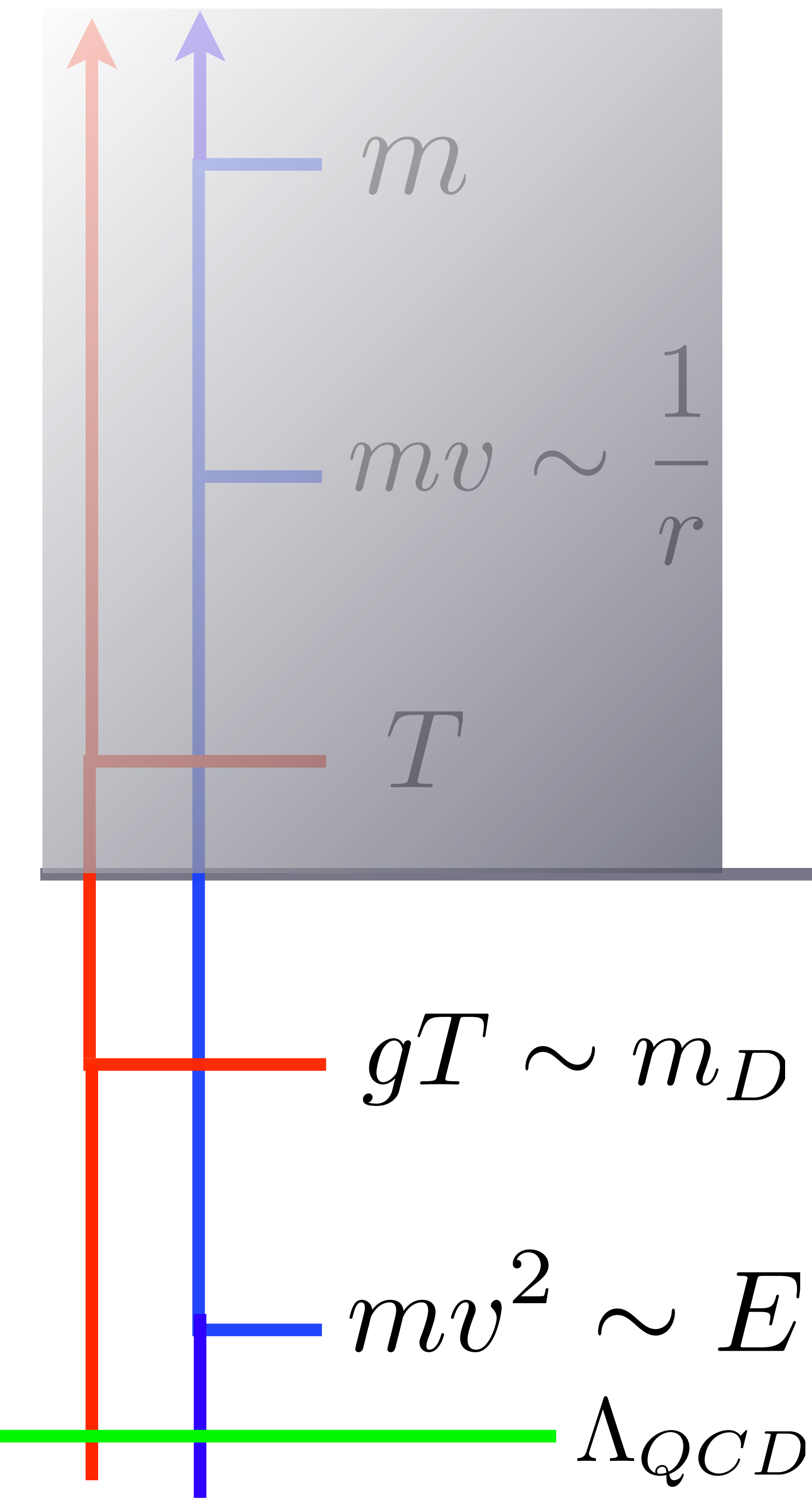
$$
\begin{aligned}
V_s(r,\mu) &= -C_F \frac{\alpha_{V_s}(1/r)}{r} \\
&= -C_F \frac{\alpha_s(1/r)}{r} \left\{ 1 + \frac{\alpha_s(1/r)}{4\pi} a_1 + \left(\frac{\alpha_s(1/r)}{4\pi} \right)^2 a_2 \right. \\
&\quad + \left(\frac{\alpha_s(1/r)}{4\pi} \right)^3 \left[\frac{16\,\pi^2}{3} C_A^3 \, {\color{blue}\ln r\mu} + a_3 \right] \\
&\quad + \left(\frac{\alpha_s(1/r)}{4\pi} \right)^4 \left[a_4^{L2} \, {\color{blue}\ln^2 r\mu} \right. \\
&\quad \left. + \left(a_4^L + \frac{16}{9} \pi^2 \, C_A^3 \beta_0 (-5 + 6\ln 2) \right) {\color{blue}\ln r\mu} + a_4 \right] \\
&\quad \left. + \qquad \cdots \qquad \right\}
\end{aligned}
$$

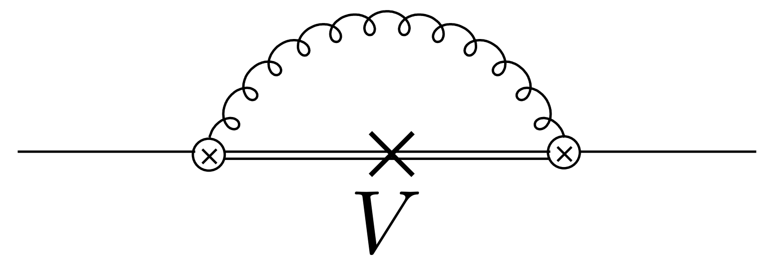
Fischler 77 Peter 97 Schröder 99 Brambilla et al. 03/08 Sumino et al. 2009 Steinhauser et al. 2009



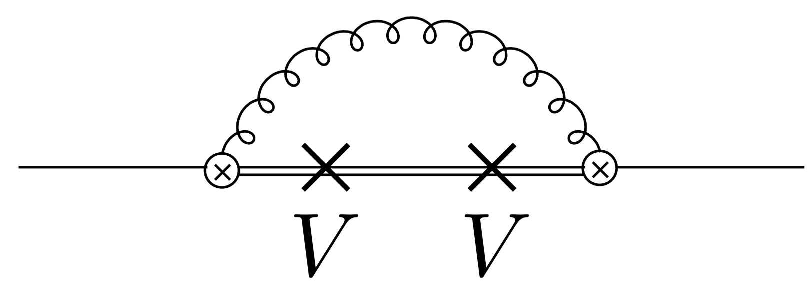
The temperature

- First thermal corrections to the potential (power law)
- Corrections appear as loops in the effective theory
- Real and imaginary parts, contributing to energy and decay width observables



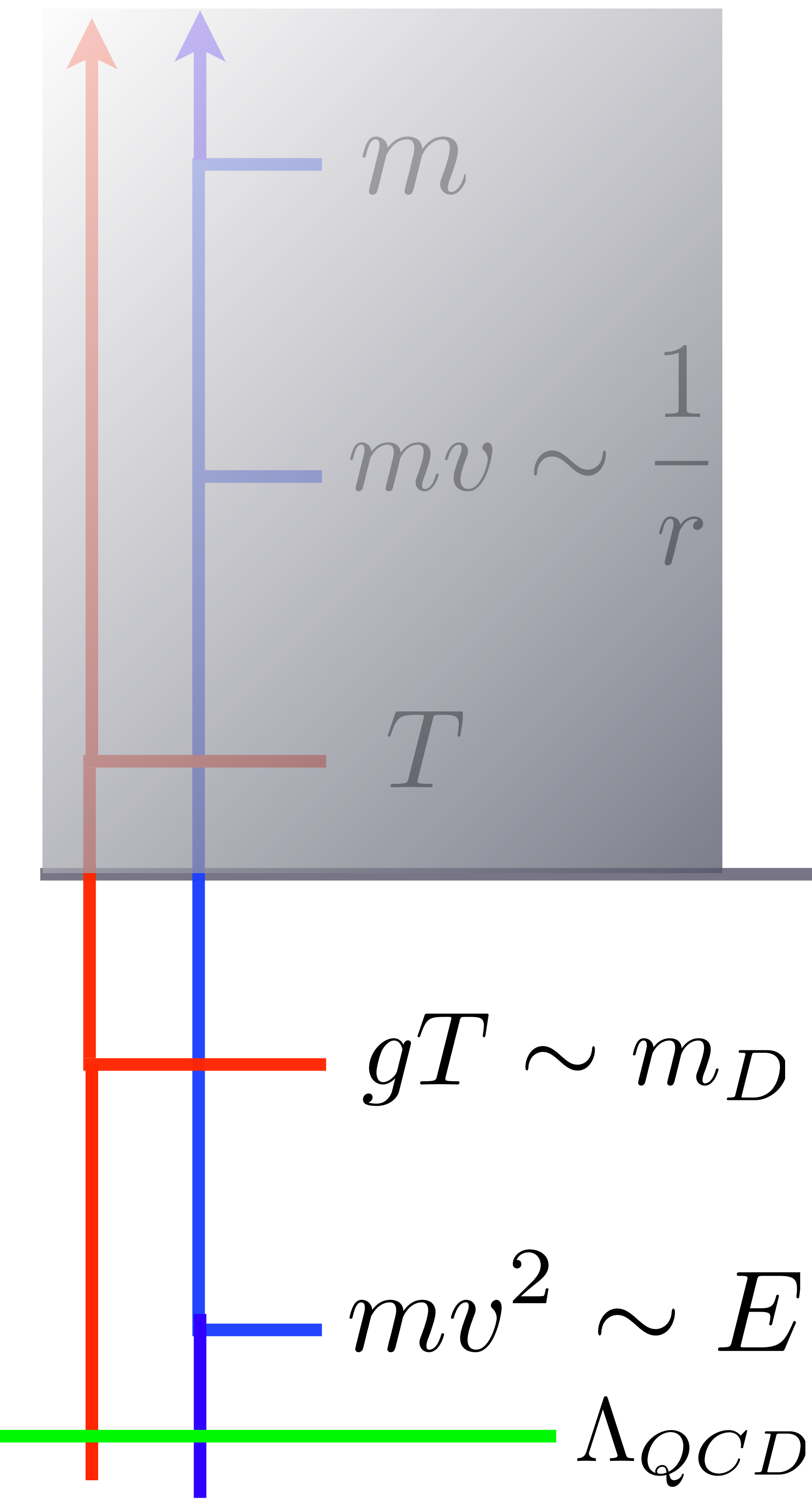


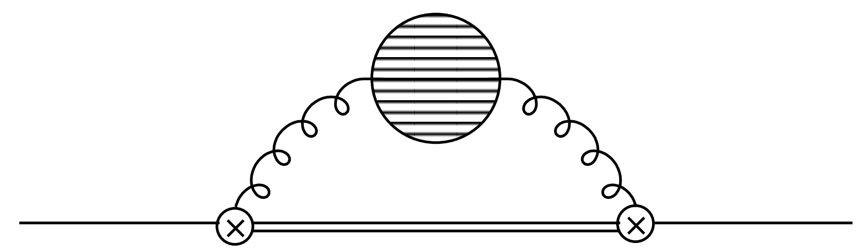
$$\mathrm{Re}\,\delta V_s(r) = \frac{\pi}{9} N_c C_F\, \alpha_s^2\, r\, T^2 \qquad \sim g^2 r^2 T^3 \times \frac{V}{T}$$



$$\mathrm{Im}\,\delta V_s(r) = -\frac{N_c^2 C_F}{6}\, \alpha_s^3\, T \qquad \sim g^2 r^2 T^3 \times \left(\frac{V}{T}\right)^2$$

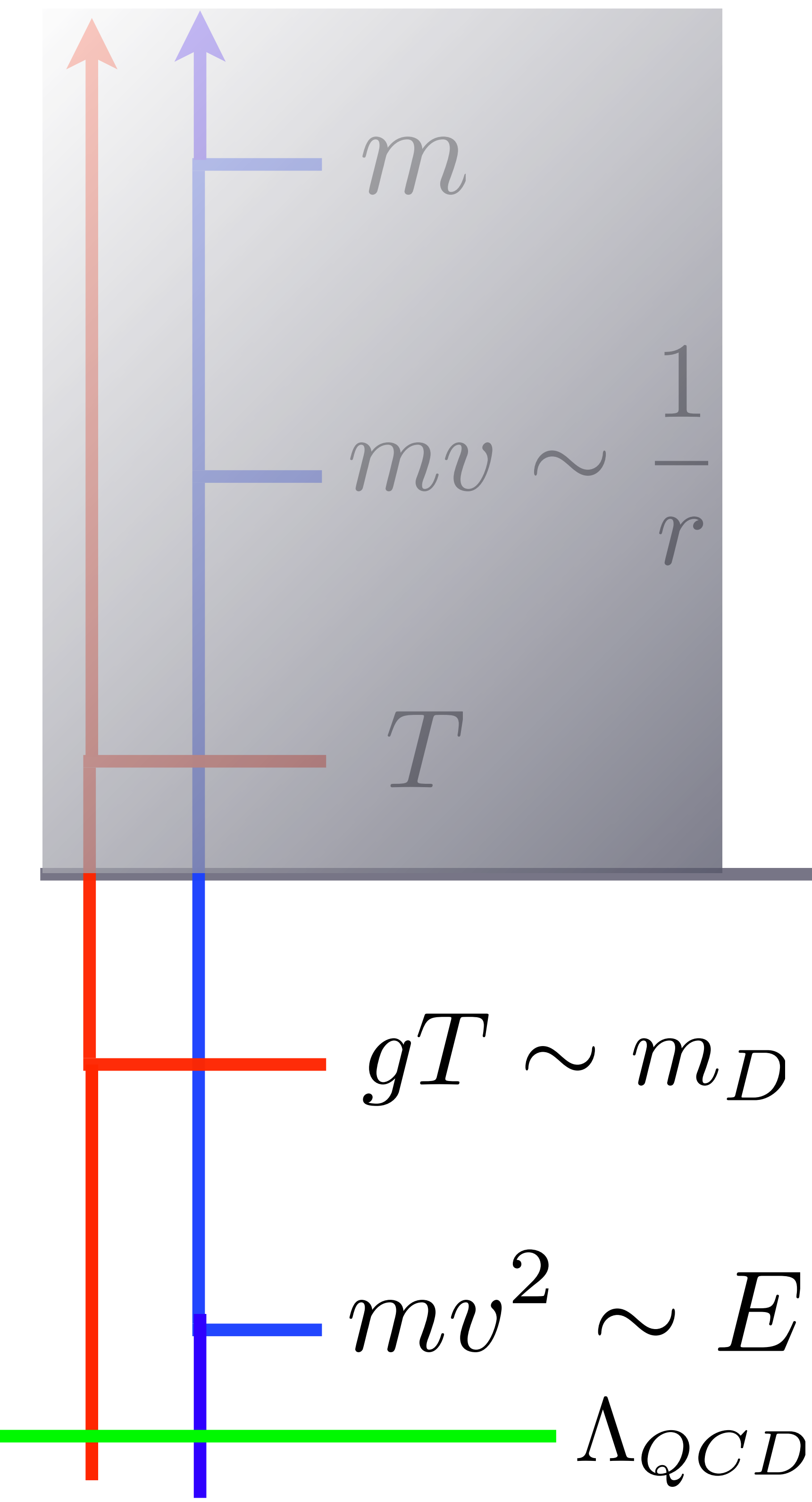
- The imaginary part correspond to singlet-to-octet thermal breakup

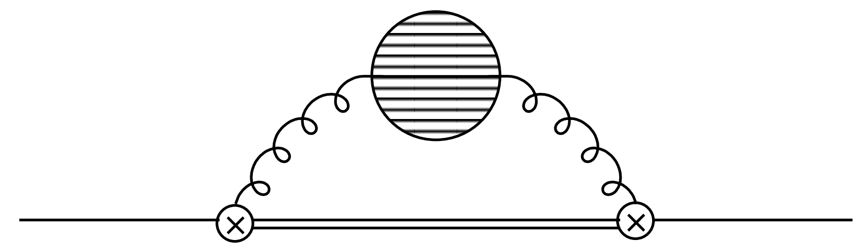




$$
\begin{aligned}
\mathrm{Re}\,\delta V_s(r) \;=\; & -\frac{3}{2}\zeta(3)\,C_F\,\frac{\alpha_s}{\pi}\,r^2\,T\,m_D^2 \\
& +\frac{2}{3}\zeta(3)\,N_c C_F\,\alpha_s^2\,r^2\,T^3 \qquad \sim g^2 r^2 T^3 \times \left(\frac{m_D}{T}\right)^2
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Im}\,\delta V_s(r) \;=\; & +\frac{C_F}{6}\alpha_s\,r^2\,T\,m_D^2\,\left(\frac{1}{\epsilon}+\gamma_E+\ln\pi \right. \\
& \left. -\ln\frac{T^2}{\mu^2}+\frac{2}{3}-4\ln 2-2\frac{\zeta'(2)}{\zeta(2)}\right) \\
& +\frac{4\pi}{9}\ln 2\;N_c C_F\,\alpha_s^2\,r^2\,T^3 \\
& \sim g^2 r^2 T^3 \times \left(\frac{m_D}{T}\right)^2
\end{aligned}
$$

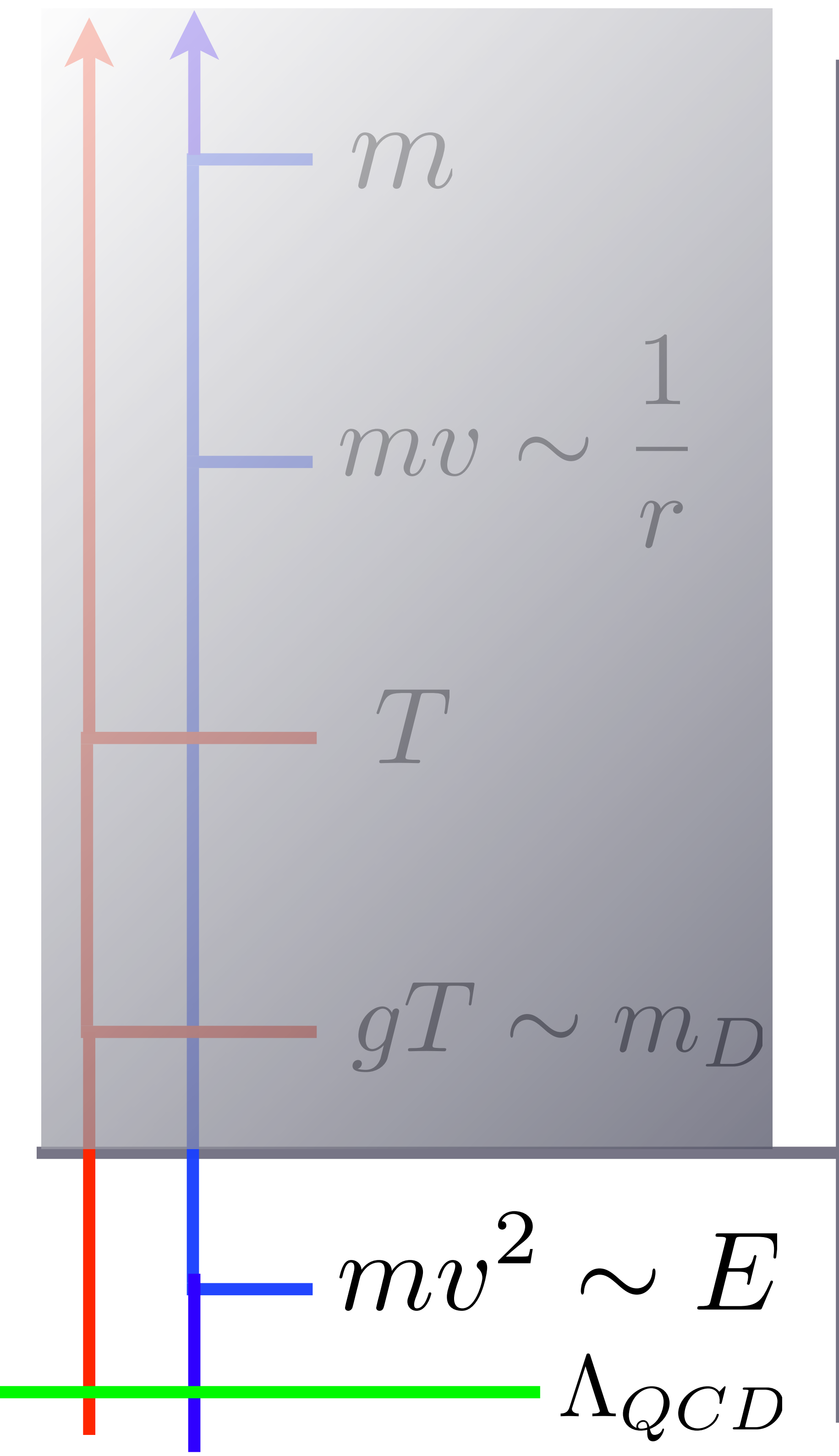




$$\mathrm{Re}\ \delta V_s(r) \;=\; -\frac{3}{2}\zeta(3)\, C_F\,\frac{\alpha_s}{\pi}\, r^2\, T\, m_D^2 + \frac{2}{3}\zeta(3)\, N_c C_F\, \alpha_s^2\, r^2\, T^3 \qquad \sim g^2 r^2 T^3 \times \left(\frac{m_D}{T}\right)^2$$

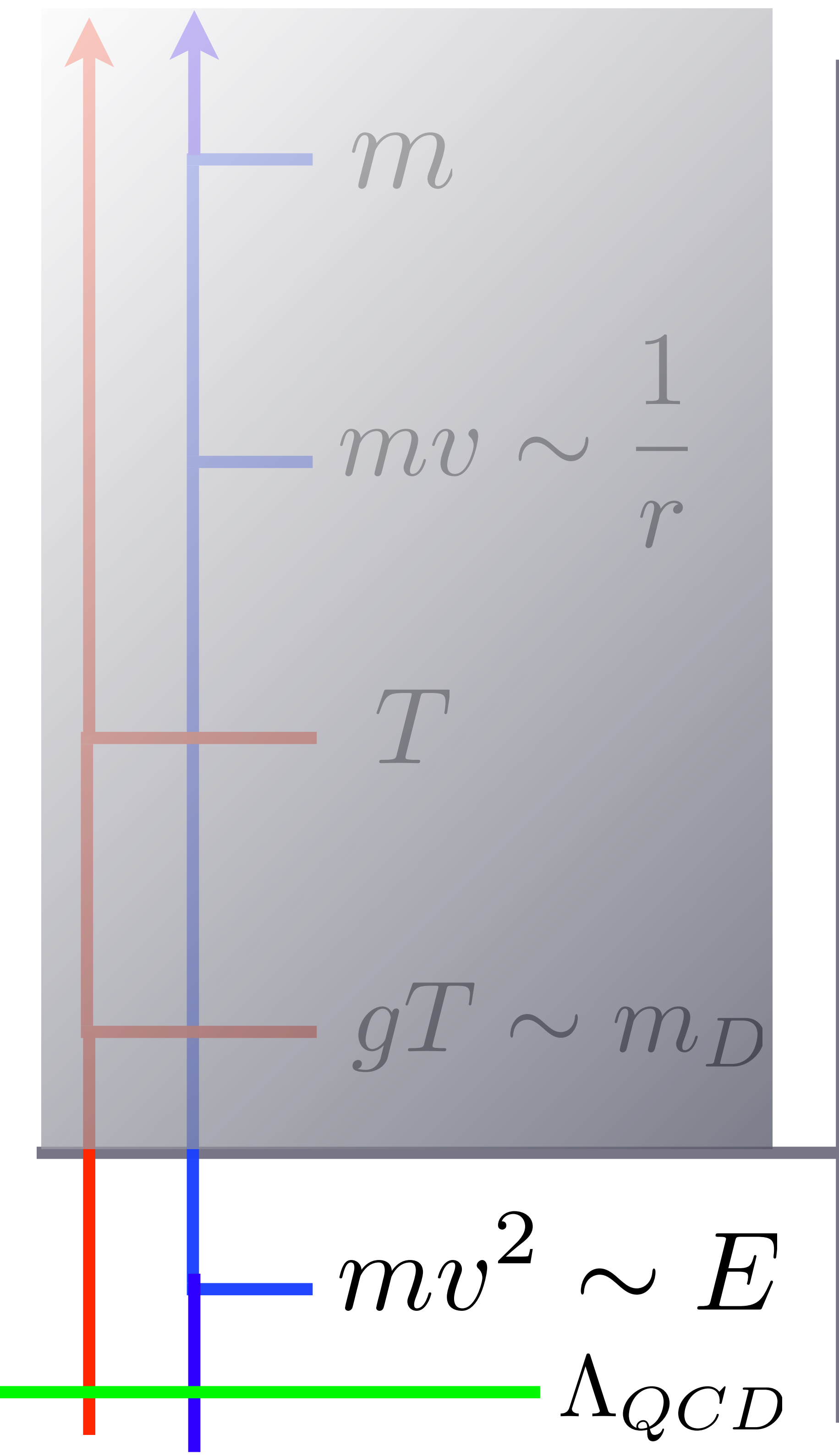
$$\mathrm{Im}\ \delta V_s(r) \;=\; +\frac{C_F}{6}\alpha_s\, r^2\, T\, m_D^2\, \left(\cancel{\frac{1}{\epsilon}} + \gamma_E + \ln\pi - \ln\frac{T^2}{\mu^2} + \frac{2}{3} - 4\ln 2 - 2\frac{\zeta'(2)}{\zeta(2)}\right) + \frac{4\pi}{9}\ln 2\; N_c C_F\, \alpha_s^2\, r^2\, T^3$$

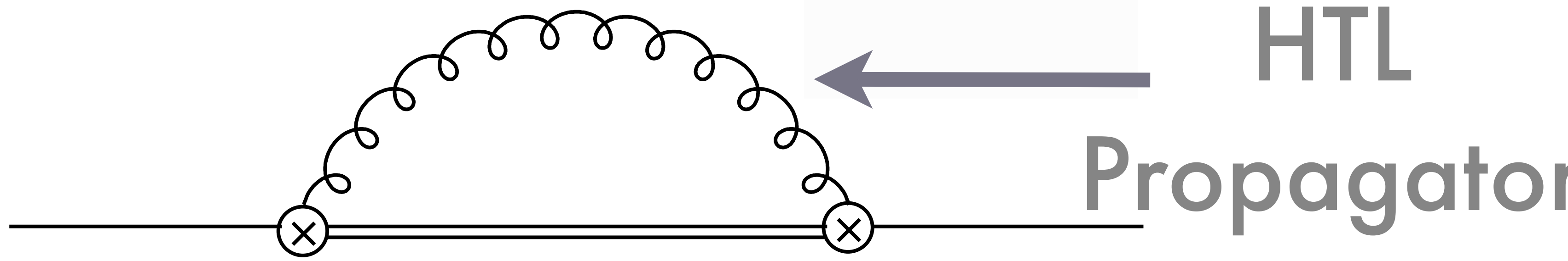
$$\sim g^2 r^2 T^3 \times \left(\frac{m_D}{T}\right)^2$$



The Debye mass

- After having integrated out the temperature Hard Thermal Loop contributions have to be resummed, giving the longitudinal gluon propagator a mass and and imaginary part
- This contribution cancels the divergence in the previous expression

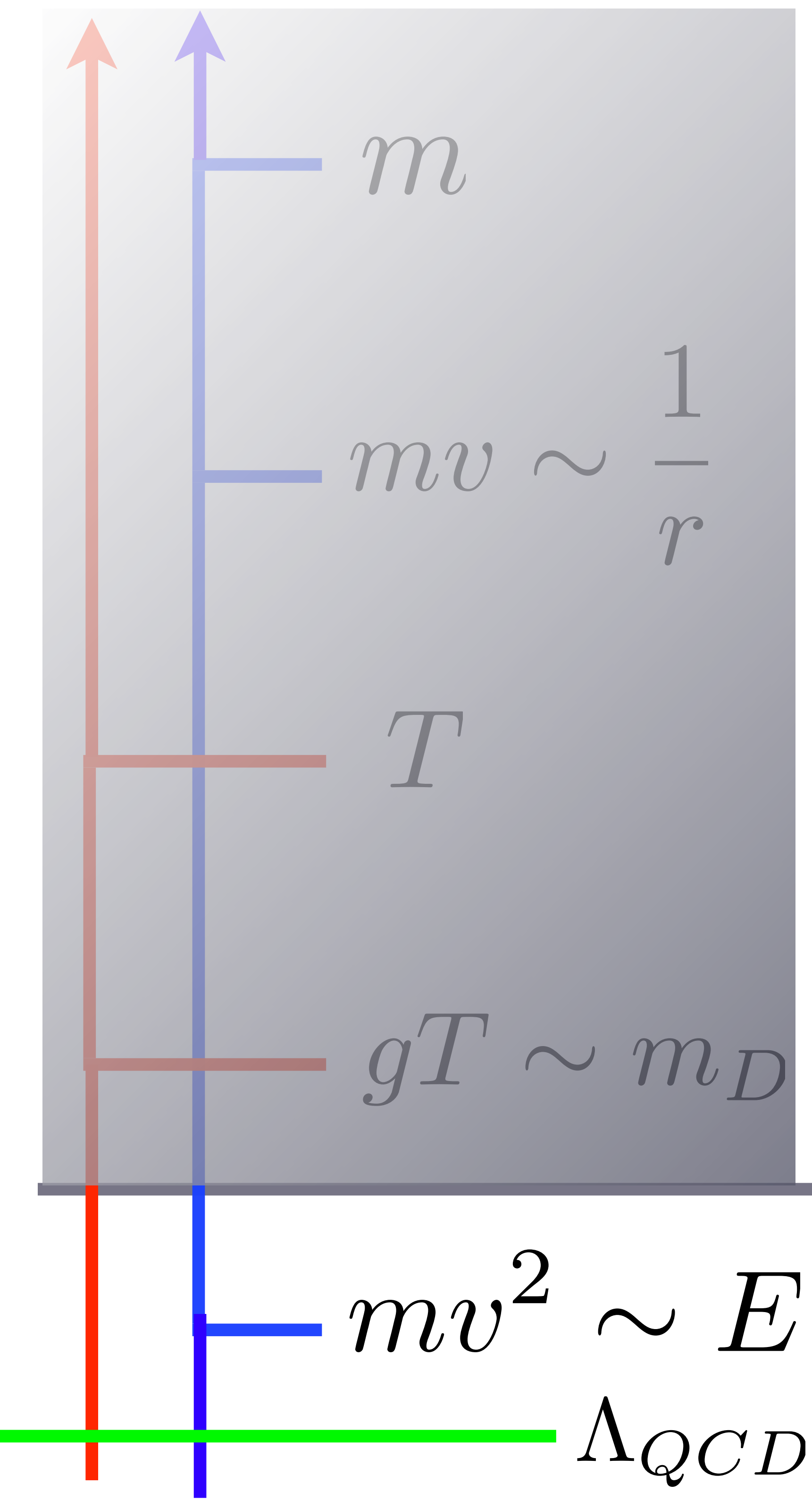


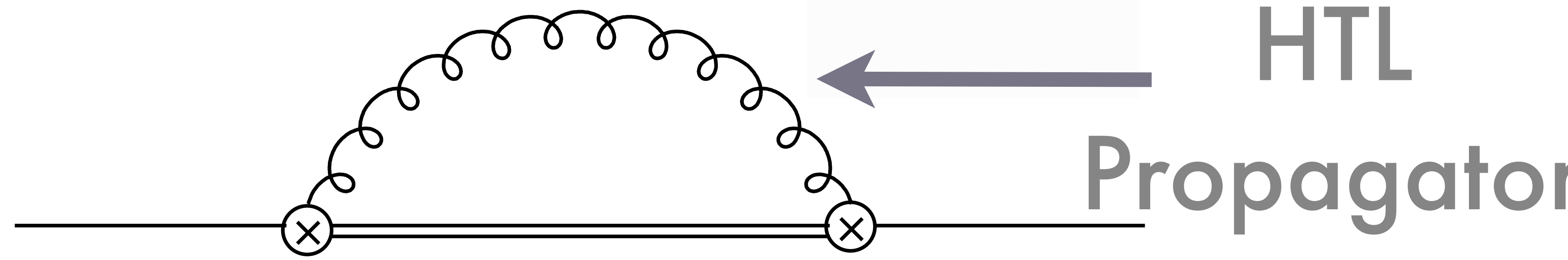


$$\mathrm{Re}\ \delta V_s(r) \ \sim\ g^2 r^2 T^3 \times \left(\frac{m_D}{T}\right)^3$$

$$\mathrm{Im}\ \delta V_s(r) \ =\ -\frac{C_F}{6}\, \alpha_s\, r^2\, T\, m_D^2 \left(\frac{1}{\epsilon} - \gamma_E + \ln \pi + \ln \frac{\mu^2}{m_D^2} + \frac{5}{3}\right)$$

- The real part is suppressed but the imaginary part indeed cancels the divergence





$$\mathrm{Re}\ \delta V_s(r) \sim g^2 r^2 T^3 \times \left(\frac{m_D}{T}\right)^3$$

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Summing up

- Divergences cancel out in the final result
- The real part of the potential is given by the Coulombic potential plus power-law thermal corrections
- The imaginary part of the static potential gives the decay width , which has two origins: singlet-to-octet breakup and Landau damping. The former is suppressed by $\left(\frac{E}{m_D}\right)^2$ vs the latter

Conclusions

- We have shown how to employ the EFT approach to deal with a problem characterized by various separated energy scales
- We have obtained new result in the intermediate regime $m \gg 1/r \gg T \gg m_D \gg E$ which could be relevant for LHC phenomenology
- We have introduced a new mechanism of thermal decay

Backup

The energy

$$
\begin{aligned}
E_0(r) \quad = \quad & -\frac{C_F\alpha_s(1/r)}{r}\Bigg\{1+\frac{\alpha_s(1/r)}{4\pi}\left[a_1+2\gamma_E\beta_0\right] \\
& +\left(\frac{\alpha_s(1/r)}{4\pi}\right)^2\left[a_2+\left(\frac{\pi^2}{3}+4\gamma_E^2\right)\beta_0^2+\gamma_E\left(4a_1\beta_0+2\beta_1\right)\right] \\
& +\left(\frac{\alpha_s(1/r)}{4\pi}\right)^3\left[\frac{16\pi^2}{3}C_A^3\ln\frac{C_A\alpha_s(1/r)}{2}+\tilde{a}_3\right] \\
& +\left(\frac{\alpha_s(1/r)}{4\pi}\right)^4\left[a_4^{L2}\ln^2\frac{C_A\alpha_s(1/r)}{2}+a_4^L\ln\frac{C_A\alpha_s(1/r)}{2}+\tilde{a}_4\right] \\
& + \qquad \cdots \qquad \Bigg\}
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& +\left(\frac{\alpha_s(1/r)}{4\pi}\right)^4\left[a_4^{L2}\ln^2\frac{C_A\alpha_s(1/r)}{2}+a_4^{L}\ln\frac{C_A\alpha_s(1/r)}{2}+\tilde{a}_4\right] \\
& + \qquad \cdots \qquad \Bigg\}
\end{aligned}
$$

Brambilla Pineda Soto Vairo **PRD60** (1999)
Brambilla Garcia Soto Vairo **PLB647** (2007)

Physical picture

- Past studies based mainly on phenomenological potential models or lattice computations of the free energy

Kaczmarek et al. 2003